

Theta Series Degree 4

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```
R.<q,t> = QQ[['q', 't']]
sum([sum([(a^4 - t*a^3*b - 30*a^2*b^2 + 5*t*a*b^3 + 25*b^4) * q^(a^2\
+ 5*b^2) for a in range(-10,10)]) for b in range(-10,10)]) + O(q\
^150)
2*q + 32*q^4 + 50*q^5 - 16*q^6 - 154*q^9 - 656*q^14 + 512*q^16 + 800*q^20 + 328*q^21 -
256*q^24 + 1250*q^25 - 2396*q^29 - 400*q^30 - 2464*q^36 + 964*q^41 - 3850*q^45 + 7024*q^46
+ 8646*q^49 + 2528*q^54 - 10496*q^56 - 8156*q^61 + 8192*q^64 - 3512*q^69 - 16400*q^70 +
12800*q^80 + 11210*q^81 + 5248*q^84 + 16624*q^86 + 8644*q^89 - 35216*q^94 - 4096*q^96 +
10000*q^100 - 11036*q^101 - 5850*q^105 - 44636*q^109 - 38336*q^116 - 3200*q^120 +
31250*q^125 + 6368*q^126 - 8312*q^129 + 35824*q^134 + 15524*q^141 - 80896*q^144 -
29950*q^145 + O(q,t)^150
```

```
R.<q> = QQ[['q']]
t = 0
lim = 15
cand_mod = sum([sum([(a^4 - t*a^3*b - 30*a^2*b^2 + 5*t*a*b^3 + 25*b\
^4) * q^(a^2 + 5*b^2) for a in range(-lim,lim)]) for b in range(-\
lim,lim)]) + O(q^150)
cand_mod_coeffs = cand_mod.list()
cand_mod
print "\n"
cand_mod_coeffs
print "\n"

cand_mod_fives = [cand_mod_coeffs[5*i] for i in range(30)]
cand_mod_fives
2*q + 32*q^4 + 50*q^5 - 16*q^6 - 154*q^9 - 656*q^14 + 512*q^16 + 800*q^20 + 328*q^21 -
256*q^24 + 1250*q^25 - 2396*q^29 - 400*q^30 - 2464*q^36 + 964*q^41 - 3850*q^45 + 7024*q^46
+ 8646*q^49 + 2528*q^54 - 10496*q^56 - 8156*q^61 + 8192*q^64 - 3512*q^69 - 16400*q^70 +
12800*q^80 + 11210*q^81 + 5248*q^84 + 16624*q^86 + 8644*q^89 - 35216*q^94 - 4096*q^96 +
20000*q^100 - 11036*q^101 + 8200*q^105 - 44636*q^109 - 38336*q^116 - 6400*q^120 +
29282*q^121 + 31250*q^125 + 50512*q^126 - 8312*q^129 + 35824*q^134 + 17608*q^141 -
39424*q^144 - 59900*q^145 + 65764*q^149 + O(q^150)
```

```
[0, 2, 0, 0, 32, 50, -16, 0, 0, -154, 0, 0, 0, 0, -656, 0, 512, 0, 0, 0, 800, 328, 0, 0,
-256, 1250, 0, 0, 0, -2396, -400, 0, 0, 0, 0, 0, -2464, 0, 0, 0, 0, 964, 0, 0, 0, -3850,
7024, 0, 0, 8646, 0, 0, 0, 0, 2528, 0, -10496, 0, 0, 0, 0, -8156, 0, 0, 8192, 0, 0, 0, 0,
-3512, -16400, 0, 0, 0, 0, 0, 0, 0, 0, 12800, 11210, 0, 0, 5248, 0, 16624, 0, 0, 8644,
0, 0, 0, 0, -35216, 0, -4096, 0, 0, 0, 20000, -11036, 0, 0, 0, 8200, 0, 0, 0, -44636, 0,
0, 0, 0, 0, 0, -38336, 0, 0, 0, -6400, 29282, 0, 0, 0, 31250, 50512, 0, 0, -8312, 0, 0, 0,
0, 35824, 0, 0, 0, 0, 0, 0, 17608, 0, 0, -39424, -59900, 0, 0, 0, 65764]
```

```
[0, 50, 0, 0, 800, 1250, -400, 0, 0, -3850, 0, 0, 0, 0, -16400, 0, 12800, 0, 0, 0, 20000,
8200, 0, 0, -6400, 31250, 0, 0, 0, -59900]
```

```
M = ModularForms(Gamma1(20), 5, prec=150)
```

```
M.sturm_bound()
```

```
len(M.basis())
```

```
121
```

```
58
```

```
B = M.basis()
```

```
the_list = [B[i].padded_list(150) for i in range(len(B))]
```

```
the_mat = matrix(the_list)
```

```
yes_mod_Bcoeffs = the_mat.solve_left(vector(cand_mod_coeffs))
```

```
yes_mod_Bcoeffs
```

```
(2, 0, 0, 32, 50, -16, 0, 0, -154, 0, 0, 0, 0, -656, 0, 512, 0, 0, 0, 800, 328, 0, 0,
-256, 1250, 0, 0, 0, -2396, -400, 0, 0, 0, 0, 0, -2464, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
```

```
yes_mod = sum([yes_mod_Bcoeffs[i] * B[i] for i in range(len(B))])
```

```
yes_mod
```

```
print "\n"
```

```
cand_mod
```

```
2*q + 32*q^4 + 50*q^5 - 16*q^6 - 154*q^9 - 656*q^14 + 512*q^16 + 800*q^20 + 328*q^21 -
256*q^24 + 1250*q^25 - 2396*q^29 - 400*q^30 - 2464*q^36 + 964*q^41 - 3850*q^45 + 7024*q^46
+ 8646*q^49 + 2528*q^54 - 10496*q^56 - 8156*q^61 + 8192*q^64 - 3512*q^69 - 16400*q^70 +
12800*q^80 + 11210*q^81 + 5248*q^84 + 16624*q^86 + 8644*q^89 - 35216*q^94 - 4096*q^96 +
20000*q^100 - 11036*q^101 + 8200*q^105 - 44636*q^109 - 38336*q^116 - 6400*q^120 +
29282*q^121 + 31250*q^125 + 50512*q^126 - 8312*q^129 + 35824*q^134 + 17608*q^141 -
39424*q^144 - 59900*q^145 + 65764*q^149 + O(q^150)
```

```
2*q + 32*q^4 + 50*q^5 - 16*q^6 - 154*q^9 - 656*q^14 + 512*q^16 + 800*q^20 + 328*q^21 -
256*q^24 + 1250*q^25 - 2396*q^29 - 400*q^30 - 2464*q^36 + 964*q^41 - 3850*q^45 + 7024*q^46
+ 8646*q^49 + 2528*q^54 - 10496*q^56 - 8156*q^61 + 8192*q^64 - 3512*q^69 - 16400*q^70 +
12800*q^80 + 11210*q^81 + 5248*q^84 + 16624*q^86 + 8644*q^89 - 35216*q^94 - 4096*q^96 +
20000*q^100 - 11036*q^101 + 8200*q^105 - 44636*q^109 - 38336*q^116 - 6400*q^120 +
29282*q^121 + 31250*q^125 + 50512*q^126 - 8312*q^129 + 35824*q^134 + 17608*q^141 -
39424*q^144 - 59900*q^145 + 65764*q^149 + O(q^150)
```

```
[yes_mod.padded_list(150)[i] - cand_mod.list()[0:150][i] for i in \
range(150)]
```

[illegible]