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As an introduction to Gröbner basis, we first shall find the points lying on the surface defined by $S = g^{-1}(0)$ where $g(x,y,z) = x^4 + y^2 + z^2$; which are closest to $(1,1,1)$. In other words, this problem is about minimising the distance function (which we shall denote by f), and to do so, we shall make use of Lagrange multipliers method. This method states that this critical points (x_0, y_0, z_0), can be found by means of the following equation

$$\nabla f(x_0, y_0, z_0) = \lambda \nabla g(x_0, y_0, z_0)$$

For our function, f(x,y,z) = $\sqrt{(x-1)^2 + (y-1)^2 + (z-1)^2}$, note that this function is non-degenerate and definite positive in $\mathbb{R}^3$ and therefore its critical points will be minimum points. Furthermore, the minimum points of f will be the same as the ones for f^2. Since it is easier to work with f^2, we will henceforth denote by f, f^2. Then the system the equations that yields

$$\begin{cases} 2(x-1) - 4\lambda x^3 = 0 \\ 2(y-1) - 2\lambda y = 0 \\ 2(z-1) - 2\lambda z = 0 \\ x^4 + y^2 + z^2 - 1 = 0 \end{cases}$$