## **Optimal Savings and Taxation**

You are asked to re-evaluate the costs and benefits of various consumption and income tax proposals prepared by a government tax panel. To do so, you consider a 2 period model where individuals earn labor income Y from working in period 1 and do not work in period 2 (retirement). Individuals choose how much to consume in each period. Savings in period 1 earn an interest rate  $r \ge 0$ . Let  $C_1$  denote the quantity of consumption in period 1 and  $C_2$  denote the quantity of consumption in period 2. Prices of consumption for both periods is normalized to one, so it is  $p_{C_1} = p_{C_2} = 1$ .

a) Write the individual's budget constraint in an economy without taxes.

b) Write the budget constraint where both labour and capital incomes are taxed at rate t

c) Write the budget constraint with just a consumption-tax of rate  $\tau$ 

d) The tax panel claims that exempting capital-income from the income-tax while retaining the income-tax on labour-income is equivalent to shifting to a consumption tax system. Prove this algebraically. (Hint: look closely at b) and c). e) Suppose that individuals have a utility function  $U(C_1, C_2) = (C_1)^{0.5} + \delta(\frac{C_2}{1+r})^{0.5}$ . Show that a consumption tax rate  $(\tau)$  does not distort consumption choices. (Hint: See notebook\_01\_optimization\_review for an illustration of optimization techniques and notebook\_02\_optimal\_saving for an example of how to solve an optimal intertemporal saving problem.

Choices are not distorted when the ratio of consumption  $C_2/C_1$  is equal in the two settings: with the consumption tax, with no taxes at all)