Riemann Sums Assignment

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Riemann Sums Assignment

Question 0

Watch the lecture video here.

Did you watch the video? [Type yes or no.]

Question 1

Approximate the area under the graph of $f(x) = 3x^2 - 9x + 5$ on the interval $[-5, 5]$ using left and right Riemann sums with $n = 25$ and $n = 50$ subintervals.

[The actual area is 300.]

Question 2

The area under the graph of $f(x) = \ln(\sin(x))$ from $x = 1$ to $x = 2$ is approximately -0.0455.

To get an idea of how big n must be to get a good approximation (say correct to four decimal places), find both the left and right Riemann sums with $n = 100$, $n = 500$, and $n = 1000$.

Question 3

The graph of $x^2 + y^2 = 25$ is a circle of radius 5 centered at the origin. From geometry, we know its area is $\pi \cdot 5^2 \approx 78.54$. We will approximate this area using Riemann sums.

Let $f(x) = \sqrt{25 - x^2}$ (the top half of the circle). Approximate the area between f and the x-axis from $x = -5$ to $x = 5$ using left and right Riemann sums with $n = 100$ subintervals.

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Now multiply this area by 2 to get an approximation for the area of the whole circle. How close are you to the correct area?

Question 4

Use Sage's sum command to evaluate the following sums.

Part a

$$\sum_{i=1}^{50} \frac{1}{i^2}$$

Part b

$$\sum_{k=10}^{100} \frac{k^3 - 3k^2}{5}$$

Part c

$$\sum_{k=1}^{n} \left(\left(\frac{k}{n} \right)^2 + \frac{k}{n} \right) \cdot \frac{1}{n}$$

[Hint: Declare both n and k to be variables.]

Question 5

Calculate the limit as $n \to \infty$ of your answer from Question 4, Part c.

Note: This limit gives the area between the x-axis and the function $f(x) = x^2 + x$ over the interval from $x = 0$ to $x = 1$, because the sum in Question 4, Part c, is the right Riemann sum with n rectangles for this function. In other words, $\int_0^1 x^2 + x \, dx = \lim_{n \to \infty} \sum_{k=1}^{n} \left(\left(\frac{k}{n} \right)^2 + \frac{k}{n} \right) \cdot \frac{1}{n}$.