# PreCalculus BC: Project Five - April 16, 2017

## *Introduction*

Irrational numbers are numbers that cannot be expressed as a ratio of two integers. Examples that you are probably familiar with are $e$ and $\pi$. Both of these numbers play important roles in mathematics. $e$ is an important constant that is related to growth and $\pi$ is useful when dealing with circles.

What you probably don't know about irrational numbers is that they can be expressed as *continued fractions*. In this project you will investigate three irrational numbers: $e$, $\pi$, and $\sqrt{2}$. Our first task will be to establish that these numbers are irrational (which actually turns out to be a tricky thing to do). After we establish that $\sqrt{2}$ is irrational, we will research the idea of a continued fraction and use the idea to develop representations for our three irrational numbers. Our third task will be to produce an algorithm that can approximate the three numbers to any number of decimals. For this part of the project, we will be using Python and SAGE to write some code that will be incorporated in your report.

## *Proving that a number is irrational*

For this part of the project, you will provide an explanation of why $\sqrt{2}$ is irrational. I'm only asking that you do a bit of research into the proof and then provide your own explanation. I'm not asking that you come up with a unique proof – that would be difficult to do.

The easiest proof available is proof by contradiction where we assume that $\sqrt{2}$ is rational and then derive a contradiction. This is exactly what I'm asking that you research and explain in your paper. Sources do not need to be cited.

## *Researching a repeated fraction*

I suggest that you start with this excellent YouTube video produced by Mathologer. The video provides a nice introduction to the idea of continued fractions and many examples are provided. As a further hint for this project, this particular YouTube channel actually has many other videos that you may find helpful.

`https://youtu.be/CaasbfdJdJg`

It might help to watch the video several times and try and work through the examples yourself.

### *An Algorithm for Approximating Irrational Numbers*

This should be where the majority of your work takes place. What we are looking for is an algorithm implemented as a function in SAGE/Python that takes as input the number of decimals requested and outputs the irrational number.

For example, if I passed in the number 4, the function would return 3.1416 for $\pi$. Please note: you can write functions for all three irrational numbers, but the report only requires that you choose one.

You can choose between $e$, $\pi$, or $\sqrt{2}$ for your irrational number. Your code should be included in your project report using the listings package. For this, you will need to add two lines to the preamble of you tex source file.

```
\usepackage{listings}
\usepackage{color}
```

Once you import those two packages, you can use the `lstlisting` environment to paste in your code. It will also help if you pass in two optional parameters that specify you are using the Python programming language. This will let LaTeX highlight and color-code your source. I have included a simple example of that below.

Listing 1: Python example

```
def myFunction(number):
    # computations go here....
    return theAnswer
```

All that I need to see is the function you have written that returns back the decimal answer.

You are welcome to copy any code you find on the Internet, as long as it does what you need it to do and that you have either commented the code to explain it, or included your own comments in paragraph form somewhere in your report. Please do not copy code that you don't understand.

### *What I will be looking for in your report*

- You have provided an appropriate introduction for the report.
- You have included a proof for the irrationality of $\sqrt{2}$.
- An explanation of continued fractions is included and you have used continued fractions to express $e$, $\pi$, and $\sqrt{2}$. For this part, I'm expecting that you display these continued fractions in an elegant way. Searching for `latex continued fraction` should help you to find the required code.

- You have working Python code that computes $n$ digits of one of the three irrational numbers: $e$, $\pi$, or $\sqrt{2}$.
- You have used the `lstlisting` environment correctly in your tex source file.