PreCalculus BC: Project Four - February 23, 2017

Introduction

Many real-world processes happen in stages. When a population is growing, each new generation represents a new stage of population growth. Compound interest is another example, where interest is paid in stages and creates a new account balance. Many things that change continuously are more easily measured in discrete stages. Temperature, which is a continuous quantity, can be measured with regular intervals producing a list of temperatures such as 67, 63, 58, 55, and so on. Although the temperature is changing continuously, we actually only care about the discrete steps from one measurement point to the next.

Recursive sequences can be used to model this types of situations. In some cases, we can get an explicit formula for a sequence from the recursion relation that defines it be finding a pattern in the terms of a sequence.

Recursive Sequences as Models

Suppose you deposit some money in an account that pays 6% interest compounded monthly. The bank has a definite rule for paying interest. At the end of each month the bank adds to your account $\frac{1}{2}\%$ (or 0.005) of the amount in your account at that time. As an equation, we have:

$$\text{amount at end of month} = \text{amount at end of last month} + 0.005(\text{amount at end of last month})$$

We can convert this into a more expressive formula, by realizing that if we let A_0 be the amount of the original deposit, A_1 is the amount at the end of the first month, A_2 is the amount at the end of the second month, and so on. A_n represents the amount at the end of n months.

$$A_n = 1.005A_{n-1}$$

We recognize this as a recursively defined sequence since it gives us the amount at each stage in terms of the amount at the preceding stage.

Daily Drug Dose - Preliminary Work not included in your report

In this project, you will construct a recursive model for the amount of a drug that is present in a patient's bloodstream. There are two important things to consider. First, the amount of drug in a patient's

bloodstream must be above a certain threshold level for the drug to be effective. Second, the amount must not exceed some maximum level beyond which the drug would do damage to the patient in some way, such as side effects of perhaps death.

Consider first a patient who takes a 50-mg pill of a certain drug every morning. It is known that the body eliminates 40% of the drug every 24 hours. Find a recursive sequence that models the amount A_n of the drug in the patient's body after each pill is taken. List out at least the first four terms of the sequence A_n. Find a formula for A_n. How much of the drug remains in the patient's body after 5 days? How much will accumulate in his system after prolonged use?

Prednisone

Prednisone is often prescribed for acute asthma attacks and suppresses the immune system. For 5 mg tablets, typical instructions are: "Take 8 tablets the first day, 7 the second, and decrease by one tablet each day until all tablets are gone." Prednisone decays exponentially in the body. This looks like regular exponential decay that we have studied previously. However, in a medical context we will develop the idea of biological half-life. The biological half-life of a substance is the time it takes for the substance to lose half of its pharmacologic activity. The Wikipedia page for biological half-life provides some additional detail, if you're interested. For prednisone, the biological half-life is one hour.

Let $x(t)$ represent the amount of prednisone (in mg) in the body at time t.

1. Write formulas involving x, for the amount of prednisone in the body:
   (a) 24 hours after taking the first dose (of 8 tablets), right before taking the second dose (of 7 tablets).
   (b) Immediately after taking the second dose (of 7 tablets).
   (c) Immediately after taking the third dose (of 6 tablets).
   (d) Immediately after taking the eighth dose (of 1 tablet).
   (e) 24 hours after taking the eighth dose.
   (f) n days after taking the eighth dose.
2. If a patient takes all the prednisone tablets as prescribed, how many days after taking the eighth dose is there less than 3% of a prednisone tablet in the patient's body?
3. A patient is prescribed n tablets of prednisone the first day, $n-1$ the second, and one tablet fewer each day until all the tablets are gone. Write a formula that represents T_n, the number of prednisone tablets in the body immediately after taking the final dose.

4. If a patient accidently takes all the prednisone tablets at once, what percentage of a prednisone tablet will be present in the patient's body? How long will it take for there to be less than 3% of a prednisone tablet in the patient's body?

What I will be looking for in your report

- Your report includes a proper introduction that explains to the reader that you will be investigating how drugs are processed in the body.
- Your report gives a brief explanation of prednisone and how it is used.
- Your report describes biological half-life and its importance in studying how drugs are processed by the human body.
- You have answered and discussed questions 1, 2, 3, and 4 as they appear in this project description. This should not appear as simply a list of answers, but instead should take on a more natural paragraph form where you walk the reader through the development of your answers.
- Strictly speaking, this report does not require any graphs, but when explaining how the concentration of prednisone changes over time, a graph can be very useful. If you decide to use a graph, it must appear in a figure environment with appropriate caption, label, and references from the text.
- Properly-formatted LaTeX style, as we have discussed in class. One of the items I will check for here is that any variables or equations are wrapped in dollar signs so that they appear typeset correctly.
- Please remember that your report's filename should follow the convention of being in all lower-case, no spaces, and should contain your last name followed by `_drug.tex`.