PreCalculus BC: Project One - October 12, 2016

Introduction

In this first project in PreCalculus BC, we won't be looking at a particularly difficult math problem. Instead, the focus should be on getting comfortable with the new tools used to create these projects.

Here is the problem we will be solving.

> A gardener has 140 feet of fencing to fence in a rectangular vegetable garden. Find a function that models the area of the garden she can fence. For what range of widths is the area greater than 825 square feet? Can she fence a garden with area 1250 square feet? Find the dimensions of the largest area she can fence.

The Process

Explore the Problem

The first step in the process of solving and writing a report about this problem is to actually solve the problem and **explore the problem space**. What does this mean? Here are a few things for you think about.

- What questions have been asked directly in the problem?
- What are the answers to those direct questions?
- In the process of finding those answers, what other questions came into your mind?
- In what ways could the problem be generalized or extended beyond what was stated? Would those extensions be useful?

Example Problem

Here is a short example to give you an idea about what exploring the problem means. Suppose that you are in charge of designing a dating website's algorithm for determining whether two people are compatible. For each person that is registered on the site, a software engineer has written an algorithm that provides you with a rectangle representing that person's character and personality. This rectangle has a length and a width along with the coordinates of the lower-left corner.

Each person's rectangle is plotted on a normal 2D plane such as the xy-axis. For any two people, there are three possibilities. First, their rectangles could have no overlap. Second, their rectangles could overlap a little bit. Third, their rectangles could be identical in which case they would completely overlap.

Your job is to take the information for each rectangle and write an algorithm that determines the amount of overlap and interpret that as how compatible two people are.

Exploring this problem means that you need to think about multiple possible solutions and consider the advantages and disadvantages of both. There isn't going to be one and only one correct answer. However, certain answers may be more easily justified than others and the quality of the justifcation is what's important.

Here are some of the things that go through my head when I think about solving this problem. These are certainly not the only things that you can investigate, but perhaps they give you some idea about what it means to explore the problem space.

- Given two rectangles, how am I going to compute the overlapping area?
- Once I have that area as a number, how can I interpret that in terms of compatibility. For example, does it matter how large or small each individual person's area is compared with the amount of overlap?
- If there is 0 overlap, but the rectangles are very close, would that have any useful meaning? In other words, could I extend the problem a little bit and consider rectangle proximity rather than only overlapping area?
- Why do we have to use a rectangle? Could other shapes be used and would they have any advantages in terms of the goals of the website?
- Can I generate some random rectangles and test my algorithm to make sure that it works the way I think it does?
- What would be the best way to illustrate compatibility in terms of area?

Waiting - time to percolate

After you spend some time with the problem and exploring the problem space, the next step involves waiting. You want to give your brain some time to think about the work you have just completed. This is important because when we are confronted with a deadline or need to finish something by a particular time, our brains aren't really in the best mode to think creatively. We need time to rest and let things percolate.

During this time, you will want to think about the problem and talk about your solution with others. Walk through all the steps and focus on what justification you had for each step. What kinds of assumptions have you made? Are they reasonable assumptions?

I cannot emphasize this step in the process enough. You simply must have time to let your brain wander and develop ideas about the problem. If you find yourself stuck, it helps to ask why you're stuck. Try to write down exactly why you feel stuck. When I get stuck, the process of thinking about why I am stuck really helps to get me unstuck.

Outlining

After a few days have passed, you will want to start and outline your solution. There are a number of different ways to do this, and you may already have a good process from a previous class. I will give you the same outlining strategy that I have used for writing mathematical papers. The basic idea is to make use of a number of notecards. Each notecard will have a single idea written on it that relates to how you solved the problem. These notecards are a way of documenting your thinking. The reason that we use notecards is that it is really easy to lay out all the cards in front of you and visualize your thought process.

Writing

Once everything seems to be clear, you can begin writing your report. The notecards will help because you already have all the information you need. This last part of writing is perhaps the most difficult because you will need to think about how to write in a clear and precise way. We will see some examples of how this looks in class and talk about ways to write clearly.

The Report

After you have written a first draft, you will bring a hardcopy to class and we will exchange papers for others to read and mark up. This is an opportunity for you to get feedback on things like typos or sentences that are not clear.

Once you have fixed any errors and re-written your report, I will show you how the report will be submitted from within SageMathCloud.

Basic Expectations

For this first project, here are the things I consider important.

- Your report is written in LaTeX and contains no spelling errors, no grammatical errors, and is written in a clear and precise way.
- You have presented your solution to the problem so that the reader clearly sees that you have considered multiple methods of solving the problem and justified your approach.

- If there are equations in your report, they are correctly typeset in a mathematical font.
- If there are graphs in your report, they are correctly titled with the axes labeled and referenced from the text of your report.

The Key

The key to these projects is not to wait until the last minute to write them. Start early. You will certainly encounter issues as you work, and if you are faced with a deadline you won't have the time to solve these issues. This way of working can be really challenging, depending on your previous experience.