PreCalculus BC: Project Three - January 12, 2017

Introduction

As we have seen, parametric equations can be powerful tools in describing a variety of curves in two-dimensional space. One example of this power is in describing the path of a projectile. At any moment in time, the position of the projectile, P, is given by

$$P : \begin{cases} x &= f(t) \\ y &= g(t) \end{cases}$$

where f and g are functions of t.

There are two parts to this project. In the first part, you will develop parametric equations that describe the flight of a simple cannonball that is launched from some angle ∂ with respect to the horizontal. In the second part, you will develop parametric equations that describe the flight of the same cannonball which is launched under the same condtions, but with a bounce off a wall that is located a distance of d meters away from the launch site.

Part One – The Cannonball

Modeling motion is one of the most important ideas in both classical and modern physics. Much of Isaac Newton's work dealt with creating a mathematical model for how objects move and interact – this was the main reason of his invention of the Calculus. Albert Einstein developed his Special Theory of Relativity in the early 1900s to refine Newton's laws of motion.

We will use coordinate geometry to model the motion of a projectile, such as a cannonball fired upward into the air. Suppose we fire a projectile into the air from ground level, with an intial speed of v_0 meters per second and an angle ∂, measured in radians, upward from the ground. If there were no gravity and no air resistance, the projectile would just keep moving indefinitely at the same speed and in the same direction. Since distance equals speed times time, the projectile would travel a distance of $v_0 t$, so its position, P, at time t would be given by the following parametric equations.

$$P : \begin{cases} x &= (v_0 \cos \partial)t \\ y &= (v_0 \sin \partial)t \end{cases}$$

But, of course, we know that gravity will pull the projectile back to ground level. Your task in part one is to modify the parametric equation given above to account for the effects of gravity. In this part, please use $g = -9.8$ meters per second per second as the gravitational constant. Be sure to completely describe the development of your new model and include an example graph showing the position over time of the projectile. This will require that you plot a parametric equation in SAGE, and we will do this in class.

Part Two - The Bounce

After launching the cannonball at an angle of θ with respect to the ground, it will collide with a wall. You will need to consider a few things. First, depending on θ, the cannonball might not even make it to the wall that is a distance of d meters away. Second, you can assume that the wall is infinitely tall, which means you would never be able to fire the cannonball over the wall. Third, you can assume the collision is purely elastic - which means no energy is lost in the impact.

Your goal will be to determine a second parametric equation that describes the flight of the cannonball after the collision. The final result will be two parametric equations. One for before the collision and one for after the collision. You must also supply the t-values for each of these two parametric equations, which will indicate how long the cannonball is in flight.

What I will be looking for in your report

- Have you justified and explained the derivation of the parametric equation in Part 1?
- Do you have the appropriate graph of the parametric equation in Part 1 and is it contained in a figure environment with a caption and referenced in the text of your report?
- Have you justified and explained the derivation of the parametric equation in Part 2?
- Do you have the appropriate graph of the parametric equation in Part 2 and is it contained in a figure environment with a caption and referenced in the text of your report? This may include the results that we will discuss in class about how to handle elastic collisions.
- Do you have the correct values for t in both parametric equations, representing what happens before and after the collision.
- Have you considered at least one potential extension of the ideas used in this project and described their potential usefulness.
- Does your report use equation formatting properly? All variables and equations should be correctly typeset.

- There should be no spacing mistakes. For example, in this sentence I have not placed a space after the period.This is bad.